P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous&ISO 9001:2015 Certified

Title of the Course: GRAPH THEORY
Semester : II

Course Code	22MA2D2	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2022-23	Year of offering : 2022-23	Year of Revision:	Percentage of Revision :

Course Objectives: The objective of this course is to understand some important classes of graph theoretic problems, properties of trees, matching, connectivity and learn some algorithms for graphs.

Course Outcomes: After successful completion of this course, students will be able to

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	understand the properties directed graphs, Euler and Hamiltonian graphs.	**K3**	**1**	**1**
CO2	understand the properties of trees.	**K3**	**7**	**1**
CO3	illustrate the properties of cut sets and cut vertices.	**K3**	**3**	**2**
CO4	detect the planarity of a graph.	**K3**	**7**	**1**
CO5	illustrate the structure of a graph as a vector space.	**K3**	**3**	**2**

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO-PO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
22MA2D2	**CO1**	**2**							**2**	
	CO2							**2**	**2**	
	CO3			**2**						**3**
	CO4							**3**	**2**	
	CO5			**3**						**3**

UNIT-I

Fundamental Concepts: What is a Graph: The Definition- Graphs as Models- Matrices and Isomorphism- Decomposition and Special graphs; Paths, cycles and trails : connection in graphs, bipartite graphs, Eulerian Circuits; Vertex degrees and counting: Directed Graphs; Hamiltonian Cycles - Necessary and Sufficient conditions.

[Sections 1.1, 1.2, 1.3, 1.4 of chapter 1and Section 7.2 of chapter 7 of Prescribed Book [1]]

UNIT-II

Trees and distance : Properties of Trees; Spanning trees in Graphs; Kruskal and Prim algorithms with proofs of correctness; Shortest paths - Dijkstra's algorithm, BFS and DFS algorithms, Application to Chinese postman problem; Trees in Computer science - rooted trees, binary trees, Huffman's Algorithm.

[Sections 2.1, 2.2, 2.3 of chapter 2 of Prescribed Book [1]]

UNIT-III

Matchings: Maximum Matchings- Hall's matching condition- Maximum bipartite matching - Augmenting path algorithm; Weighted bipartite matching - Hungarian algorithm and solving the assignment problem; Tutte's theorem.

[Sections 3.1, 3.2, 3.3 of Chapter 3 of Prescribed Book [1]]

UNIT-IV

Connectivity and Paths: Connectivity; 2-connected graphs; Menger's theorem; Network flow problems - Ford-Fulkerson labelling algorithm, Max-flow Min-cut Theorem.

[Sections 4.1, 4.2, 4.3 of chapter 4 of Prescribed Book [1]]

UNIT-V

Coloring of Graphs: Definition and Examples; Upper Bounds- Greedy coloring algorithm- Brooks' theorem; Graphs with large chromatic number; Extremal problems and Turan's theorem.

Planar Graphs: Planar graphs; Dual graphs; Euler's formula; Preparation for Kuratowski's Theorem; Coloring of Planar Graphs-Five Color Theorem; Four Color Problem.

[Sections 5.1, 5.2 of Chapter 5 & Sections 6.1, 6.2, 6.3 of chapter 6 of Prescribed Book [1]]

PRESCRIBED BOOK : [1] “Introduction to Graph Theory”, Douglas B. West, Second Edition, Prentice Hall, 2001.

REFERENCE BOOKS:

1. "Graph Theory", R. Diestel, Second Edition, Springer, 2017.
2. "Graph Theory with Applications to Engineering and Computer Science", NarsinghDeo, Prentice-Hall, 2001.

Course has Focus on :Foundation(Elective paper)

Websites of Interest: 1. www. nptel.ac.in
2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

Second Semester

GRAPH THEORY – 22MA2D2

Time: 3 hours **Max. Marks: 70**

SECTION-A

Answer all questions. **(5x4=20)**

1 a) Define (i) Adjacency matrix and (ii) incident matrix of a graph with examples.(CO1, L1)

(OR)

b) State and prove Hand shaking lemma. (CO1, L1)

2 a) Define (i) Spanning tree and (ii) Weighted graph with examples. (CO2, L2)

(OR)

b) Define (i) diameter and (ii) eccentricity of a graph with examples. (CO2, L2)

3 a) Define (i) Maximal Matching (ii) Augmenting path. Write Hall's matching condition. (CO3, L2)

(OR)

b) Define (i) Edge connectivity (ii) Vertex connectivity of a graph. Write Tutte's condition. (CO3, L2)

4 a) State and prove expansion lemma. (CO4, L2)

(OR)

b) Prove that two blocks in a graph share at most one vertex. (CO4, L2)

5 a) Define chromatic number. Find the chromatic number of C_5. (CO5, L1)

(OR)

b) Define a Planar graph. Prove that K_5 and $K_{3,3}$ are not planar. (CO5, L1)

SECTION-B

Answer all questions. All questions carry equal marks. **(5X10=50)**

6 a) Show that a graph is Eulerian if and only if it has at most one non trivial component and its vertices all have even degree. (CO1, L2)

(OR)

b) Show that the minimum number of edges in a connected graph with n vertices is n-1. (CO1, L2)

7 a) For a n vertex graph G, show that the following are equivalent.

(i) G is connected and has no cycles.

(ii) G is connected and has n-1 edges.

(iii)G has n-1 edges and no cycles.

(iv) For $u,v \in V(G)$,G has exactly one u,v-path. (CO2, L2)

(OR)

b) Explain Kruskal's algorithm with an example. (CO2, L2)

8 a) State and prove Hall's theorem. (CO3, L3)

(OR)

b) Write Augmenting path algorithm and show that by applying augmenting path algorithm to a bipartite graph produces a matching and a vertex cover of equal size. (CO3, L3)

9 a) Show that a graph G having atleast three vertices is 2-connected if and only if for each pair $u,v \in V(G)$, there exists internally disjoint u,v-paths in G. (CO4, L3)

(OR)

b) If x, y are vertices of a graph G and $x,y \notin E(G)$, then prove that the minimum size of an x,y-cut equals the maximum number of pairwise internally disjoint x,y-paths. (CO4, L3)

10 a) State and prove Brook's theorem. (CO5, L3)

(OR)

b) If a connected plane graph G has exactly n vertices, e edges and f faces, then prove that n-e+f=2. (CO5, L3)

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