P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: FUZZY SETS AND FUZZY LOGIC
Semester : III

Course Code	22MA3D5	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2020-21	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision :5%

Course Objective : The objective of the course is to acquire knowledge on basic concepts of fuzzy sets, fuzzy relations, fuzzy logic and their applications.

Course Outcomes: After successful completion of this course, students will be able to

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	differentiate crisp sets and fuzzy sets.	K3	1	1
CO2	define various operations on Fuzzy sets.	K3	3	1
CO3	understand the concept of fuzzy arithmetic.	K3	1	1
CO4	determine the difference between crisp relations and fuzzy relations.	K3	3	2
CO5	understand the concept of fuzzy logic and fuzzy inference systems.	K3	1	1

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
22MA3D5	CO-PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
	CO1	2							2	
	CO2			2					2	
	CO3	2							2	
	CO4			3						3
	CO5	2							2	

UNIT– I

From Classical (Crisp) sets to Fuzzy sets: Introduction, Crisp Sets: An overview, Fuzzyset: Basic types, Fuzzy sets: Basic Concepts, Characteristics and significance of the paradigm shift.

Fuzzy sets versus Crisp sets: Additional Properties of α-cuts, Representations of Fuzzy sets, Extension principle for Fuzzy sets (Chapters 1 and 2 of prescribed book [1]).

UNIT - II

C **Operations on Fuzzy sets**: Types of Operations, Fuzzy Compliments, Fuzzy Intersections: t-Norms, Fuzzy unions: t-Co norms, Combinations of operations, Aggregation Operations (Chapter 3 of prescribed book [1]).

UNIT – III

Fuzzy Arithmetic: Fuzzy Numbers, Linguistic Variables, Arithmetic Operations on Intervals, Arithmetic Operations on Fuzzy numbers, Lattice of fuzzy numbers, Fuzzy equations (Chapter 4 of prescribed book [1]).

UNIT –IV

Fuzzy Relations: Crisp versus fuzzy relations, Projections and Cylindric Extensions, Binary Fuzzy Relations, Binary Relations on a Single set, Fuzzy Equivalence Relations, Fuzzy Compatibility Relations, Fuzzy Ordering Relations, Fuzzy Morphisms, Sup – i Compositions of Fuzzy Relations, Inf- ω_i Compositions of fuzzy Relations.
(Chapter 5 of prescribed book [1])

UNIT – V

Fuzzy Logic: Classical Logic: an Over View, Multivalued Logics, Fuzzy Propositions, Fuzzy Quantifiers, Linguistic Hedges, Inference from conditional Fuzzy Propositions, Inference from conditional and qualified propositions, Inference from Quantified propositions. (Chapter 8 of prescribed book [1])

PRESCRIBED BOOK:

1. Klir G.J. and YUAN B, Fuzzy sets and Fuzzy Logic, Theory and Applications**,** Prentice - Hall of India Pvt. Ltd., New Delhi.,(2001).

REFERENCE BOOK :

1. Zimmermann H.J, Fuzzy set Theory and its Applications**,** Allied Publishers(1996).

Course has Focus on : Foundation (Elective Paper)

Websites of Interest : 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An Autonomous college in the jurisdiction of Krishna University)

M.Sc. Mathematics

Third Semester

FUZZY SETS AND FUZZY LOGIC -22MA3D5

Time:3 hours **Max. Marks: 70**

SECTION A

Answer all questions. **(5x4=20)**

1 a) Let A, B be fuzzy sets defined on a universal set X. Prove that $|A|+|B|=|A\cup B|+|A\cap B|$. (CO1, K2)

(OR)

b) State and prove second decomposition theorem. (CO1, K2)

2 a) State axioms of fuzzy complements. Prove that a functions $C:[0,1]\to[0,1]$ satisfies axioms C_2 and C_4. Then C also satisfy axioms C_1 and C_3 . (CO2, K3)

(OR)

(b) State and prove second characterization theorem of fuzzy complement. (CO2, K3)

3 a) Let R denote the set of all fuzzy numbers. For any A, B $\in$R, prove that (i) MIN(A, B) = MIN(B, A) and (ii) MAX(A, B) = MAX (B, A) (CO3, K3)

(OR)

b) Let A and B be fuzzy numbers. Is A-B a solution of λ + B = A? Justify your answer. (CO3, K3)

4 a) For any fuzzy relation R on X^2 prove that the fuzzy relation $R_{T(i)}=\bigcup_{n=1}^{\infty}R^{(n)}$ is the i-transitive closure of R. (CO4, K2)

(OR)

b) Explain homomorphism of fuzzy sets with example. (CO4, K2)

5 a) Define Unconditional and Unqualified propositions and explain these propositions using an example. (CO5, K2)

(OR)

b) Define conditional and qualified propositions and explain these propositions using an example. (CO5, K2)

SECTION B

Answer all questions. **(5x10=50)**

6 (a) Let A,B be two fuzzy sets of a universal set X. The difference of A and B is defined by $A-B=A\cap\overline{B};$ and the symmetric difference of A and B is defined by $A\Delta B=(A-B)\cup(B-A).$ Prove that (i) $(A\Delta B)\Delta C=A\Delta(B\Delta C).$

(ii) $A\Delta B\Delta C=(\overline{A}\cap\overline{B}\cap C)\cup(\overline{A}\cap B\cap\overline{C})\cup(A\cap\overline{B}\cap\overline{C})\cup(A\cap B\cap C).$ (CO1, K2)

(OR)

(b) Let $f:X\to Y$ be an arbitrary using function. Then prove that for any $A\in f(X)$ and all $\alpha\in[0,1]$ the following properties of f fuzzified by the extension principle hold

$(i)^{\alpha+}[f(A)]=f(^{\alpha+}A)$

$(ii)^{\alpha}[f(A)]\geq f(^{\alpha}A)$ (CO1, K2)

7 (a) State and prove every fuzzy complement has atmost one equilibrium. (CO2, K3)

(OR)

(b) Prove that the standard fuzzy union is the only the idempotent t- norm. (CO2, K3)

8 a) Let *E [+, -, ., /] and let A, B denote continuous fuzzy numbers. Then prove that the fuzzy set (A*B)(z)= $\sup_{z=x*y}\min[A(x),B(y)] forall z\in R$ is a continuous fuzzy number.

(CO3, K4)

(OR)

(b) For any fuzzy numbers A,B,C prove the following

(i) MIN [A, MAX (A.B)] = A

(ii) MIN [A, MIN[B,C]] = MIN [MIN[A,B,],C]. (CO3, K4)

9 (a) Let R(X, X) and R(Y,Y) be a fuzzy relation defined on the sets X={a, b, c, d} and Y={α, β, γ} respectively are given below:

$$R=\begin{array}{c|cccc} & a & b & c & d\\ a & 0 & .5 & 0 & 0\\ b & 0 & 0 & .9 & 0\\ c & 1 & 0 & 0 & .5\\ d & 0 & .6 & 0 & 0\end{array}\qquad Q=\begin{array}{c|ccc} & \alpha & \beta & \gamma\\ \alpha & .5 & .9 & 0\\ \beta & 1 & 0 & .9\\ \gamma & 1 & .9 & 0\end{array}$$

Define h: $X\to Y$ be $h(a)=h(b)=\alpha, h(c)=\beta, h(\alpha)=\gamma$, prove that h: $X\to Y$ is a homomorphism. (CO4, K4)

(OR)

b) Let a, b, d Є[0,1]. Prove the following:

(a) $i(a,b) \leq d$ if and only if $W_i(Q,d) \geq b$

$(b) W_i(W_i(a,b),b) \geq a.$

$(c) W_i(i(a,b),d) = W_i(a, W_i(b,d)).$

$(d) a \leq b \Rightarrow W_i(Q,d) \geq W_i(b,d) and W_i(d,a) \leq W_i(d,b)$ (CO4, K4)

10 a) Let a fuzzy proposition of the form be given, where S is the identity function (i.e., S stands for true), and let a fact be given in the form " χ is A'," where $\sup_{x:A(x)=a} A'(x) = A'(x_0)$ for all aЄ[0,1] and some x_0 such that A(x_0) =a. Then show that the inference "y is B'" obtained by the method of truth value restrictions is equal to the one obtained by the generalized modus ponens , provided that we use the same fuzzy implication in both inference methods. (CO5, K4)

(OR)

b) Let sets of values of variables x and y be X={x_1, x_2, x_3} and Y={y_1,y_2}, respectively. Assume that a proposition "if x is A, then y is B" is given, where A=.5/x_1+1/x_2+.6/x_3 and B=1/y_1+.4/y_2. Then given a fact expressed by the proposition "x is A'," where A'=.6/x_1+.9/x_2+.7/x_3, use the generalized modus ponens to derive a conclusion in the form "y is B''. (CO5, K4)

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