PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

22MA4D4 : SEMIGROUPS
Semester : IV

Course Code	22MA4D4	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2021-22	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision :5%

Course Objective : The objective of the course is to acquire knowledge on basic concepts of Semigroups, Semilattices, congruences, Regular Semigroups, Simple Semigroups and Completely 0- Semisimple groups.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	know the basic definitions in semigroup theory
CO2	construct new semigroups using congruences
CO3	understand the basic properties of Green's relations, regular semigroups
CO4	understand the definitions of 0-simple semigroups and the proofs of some of the main theorems in this section
CO5	understand the concepts of congruences on completely 0-simple semigroups, free semi groups

Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO1	2	0	0	0	0	0	0
CO2	0	0	0	0	0	0	3
CO3	2	0	0	0	0	0	0
CO4	0	0	0	0	0	0	3
CO5	3	0	0	0	0	0	0

UNIT – I

Basic Definitions, Monogenic Semigroups, Ordered Sets, Semilattices and Lattices, Binary Relations, Equivalences. (Sections 1 to 4 of Ch. I of prescribed book [1])

UNIT –II

Congruences, Free Semigroups, Ideals and Rees Congruences, Lattices of Equivalences and Congruences. (Sections 5 to 8 of Ch. I of prescribed book [1])

UNIT – III

Introduction, The equvivalences L ,R , H, J and D , The structure of D **-** Classes, Regular Semigroups. (Chapter II of prescribed book [1])

UNIT – IV

Introduction, Simple and 0 – Simple Semigroups, Principle Factors, Rees's Theorem, Primitive Idempotents. (Sections 1 to 3 of Chapter III of prescribed book [1])

UNIT – V

Congruences on Completely 0 – Simple semigroups, The Lattice of Congruences on a Completely 0 – Simple Semigroup, Finite Congruence- Free Semigroups. (Sections 4 to 6 of Chapter III of prescribed book [1])

PRESCRIBED BOOK:

1. Howie, J.M, An Introduction to Semigroup Theory*,* Academic Press(1976).

REFERENCE BOOKS:

1. Clifford A.H, Preston G.B, The Algebraic Theory of Semigroups, American Mathematical Society(1961).

Course has Focus on : Foundation (Elective Paper)

Websites of Interest: 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

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M.Sc. Mathematics
Fourth Semester
22MA4D4 - SEMIGROUPS

Time:3 hours **Max. Marks: 70**

SECTION - A

Answer all questions. **(5x4=20)**

1 a) Define a Semigroup and a Semilattice. (CO1, L1)

(OR)

b) Define an equivalence relation and give an example. (CO1, L1)

2 a) Define a congruence relation and give an example. (CO2, L2)

(OR)

b) Define a Free semigroup and give an example. (CO2, L2)

3 a) Explain briefly about D-Classes. (CO3, L1)

(OR)

b) Define Regular semigroup and give an example. (CO3, L1)

4 a)(Define 0- simple semi group and give an example. (CO4, L1)

(OR)

b) Define a congruence Free semigroup and give an example. (CO4, L1)

5 a) Define completely 0-simple semigroup and give an example. (CO5, L2)

(Or)

b) Define Lattice of congruences and give an example. (CO5, L2)

SECTION-- B

Answer the following questions. All questions carry equal marks. **(5X10=50)**

6 (a) Show that a semi group S with zero is a o- group if and only if ($\forall$ a Є S/{0})
aS = Sa= S . (CO1, L2)

(OR)

(b) If R is any binary relation on a set X, then prove that $R^e = [R \cup R^{-1} \cup 1_x]^{\infty}$.

(CO1, L2)

7 (a) If R is a relation on a semi group S, then show that $R^{\#} = (R^c)^e$. (CO2, L3)

(OR)

(b)Show that a modular lattice is semi modular. (CO2, L3)

(Turn Over)

8 (a) If H is an H- class in a Semi group S then show that either $H^2 \cap H = \phi$ or $H^2 = H$ and H is a subgroup of S. (CO3, L3)

(OR)

(b) If H and K are two group H – classes in the same D – class then show that H and K are isomorphic. (CO3, L3)

9 (a) If M is a 0- minimal ideal of S then show that either $M^2 = 0$ or M is a 0- simple semi group. (CO4, L4)

(OR)

(b) Show that a 0-Simple semi group is completely 0- Simple if and only if it contains a primitive idempotent. (CO4, L4)

10 (a) If S is a finite congruence free semi group with out zero and if $|S| > 2$ then prove that S is a simple group. (CO5, L3)

(OR)

(b) If ρ and σ are proper congruences on $S = \mu^0[G: I, \wedge: \rho]$ then show that

$$\rho \cap \sigma = [N_\rho \cap N_\sigma, \rho_1 \cap \sigma_1, \rho_\wedge \cap \sigma_\wedge]$$

$$\rho \vee \sigma = [N_\rho . N_\sigma, \rho_1 \vee \sigma_1, \rho_\wedge \vee \sigma_\wedge]$$ (CO5, L3)

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