PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

22MA4D5 : OPERATOR THEORY
Semester : IV

Course Code	22MA4D5	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2023-24	Year of offering : 2023-24	Year of Revision: ----	Percentage of Revision :--

Course Objectives: To develop skills and to acquire knowledge on advanced concepts in Category theorem, Open mapping theorem, Closed Graph theorem, Banach's theorem and it's applications, Spectral theory etc.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Characterize the category of normed spaces using Category theorem and differentiate weak and pointwise convergence of linear operators
CO2	Apply Banach's Theorem to Linear Equations, differential Equations and Integral Equations.
CO3	Demonstrate Spectral properties of Bounded Linear Operators
CO4	Understand Banach algebras, Demonstrate spectral properties of compact linear operators
CO5	Study Operator equations involving Compact linear operators

Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO1	3	0	0	0	0	0	0
CO2	0	0	0	0	0	0	3
CO3	3	0	0	0	0	0	0
CO4	2	0	0	0	0	0	0
CO5	0	0	0	0	0	0	3

UNIT- I

Category Theorem, Uniform Boundedness Theorem, Strong and Weak Convergence, Convergence of Sequences of Operators and Functionals, Open Mapping Theorem, Closed Linear Operators, Closed Graph Theorem.
(Sections 4.7, 4.8, 4.9, 4.12 and 4.13 of Chapter 4).

UNIT- II

Banach Fixed Point Theorem, Application of Banach's Theorem to Linear Equations, Application of Banach's theorem to differential Equations, Application of Banach's theorem to Integral Equations. (Chapter 5)

UNIT –III

Spectral theory in finite dimensional normed spaces, Basic concepts, Spectral properties of Bounded Linear Operators, Further properties of resolvent and Spectrum.
(Sections 7.1 to 7.4 of Chapter -7)

UNIT –IV

Banach Algebras, Further properties of Banach Algebras, Compact linear operators on Normed spaces, Further properties of compact linear operators, Spectral properties of Compact linear operators on Normed spaces.(Sections 7.6, 7.7 of Ch. 7 & Sections 8.1 to 8.3 of Ch.8)

UNIT –V

Further Spectral properties of Compact linear operators, Operator equations involving Compact linear operators, Further Theorems of Fredholm type, Fredholm alternative.
(Sections 8.4 to 8.7 of Chapter -8)

PRESCRIBED BOOK: Erwin Kreyszig, *Introductory Functional analysis with Applications*, John Wiley & Sons(1978).

REFERENCE BOOK: M. Thamban Nair, *Functional Analysis- A First Course*, PHI(2002).

Course has Focus on : Foundation

Websites of Interest : 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

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M.Sc. Mathematics
Fourth Semester
22MA4D5 - OPERATOR THEORY

Time:3 hours **Max. Marks: 70**

SECTION – A

Answer all questions. **(5x4=20)**

1 a) Let $\{x_n\}$ be a sequence in a normed space and dim X< ∞, then show that $\{X_n\}$ is weakly convergent implies $\{x_n\}$ is strongly convergent. (CO1, L2)

(OR)

b) Define a closed linear operator and give an example. (CO1, L2)

2 a) Prove that a contraction mapping on a metric space X is continuous. (CO2, L2)

(OR)

b) Explain Jacobi iteration method. (CO2, L2)

3 a) Prove that the resolvent set ρ (T) of a bounded linear operator T on a complex Banach space X is open and hence show that the spectrum σ (T) is closed . (CO3, L3)

(OR)

b) Show that the spectrum σ (T) of a bounded linear operator T: X $\rightarrow$X on a complex Banach space X is compact and lies in the disk given by $|\lambda| \leq \|T\|$. (CO4, L3)

4 a) Define a normed algebra and Banach algebra with examples.

(OR)

b) Prove that every compact linear operator is bounded and continuous. (CO4, L3)

5 a) Let T be a compact linear operator on Banach space X. Then prove that every non zero spectral value of T is an eigen value of T. (CO5, L2)

(OR)

b) let T: X $\rightarrow$X be a compact linear operator on a normed space X. Then prove that if T has nonzero eigen values, every one of them must be an eigen value of T. (CO5, L2)

SECTION - B

Answer Five Questions. All Questions Carry Equal Marks. **(5×10=50M)**

6 a) State and prove unifrom boundedness theorem. (CO1, L4)

(OR)

b) State and prove open mapping theorem. (CO1, L4)

7a) State and prove Banach fixed point theorem. (CO2, L3)

(OR)

b) State and prove Picards existence and uniqueness theorem. (CO2, L3)

8a) Let T $\in$ B(X, X), where X is Banach space. If $\|T\| < 1$ then Prove that $(1 - T)^{-1}$ exists as a bounded linear operator on the whole space X and $(1 - T)^{-1} = \sum_{k=0}^{\infty} T^k$. (CO3, L3)

(OR)

b) State and prove spectral mapping theorem for polynomials. (CO3, L3)

9 a) Let A be a complex Banach algebra with identity e. If x in A satisfies $\|x\| < 1$, then prove that e-x is invertible and $(e - x)^{-1} = e + \sum_{j=1}^{\infty} x^j$ (CO4, L3)

(OR)

b) Show that the set of all eigen values of a compact linear operator T on a normed linear space X is countable. (CO4, L3)

10 a) Let T :X→X be a compact linear operator on a normed space X and $\lambda \neq 0$. Then prove that $Tx - \lambda x = 0$ has a solution x if and only if y is such that f(y) = 0, for all $f \in X^1$ satisfying $T^X f - \lambda f = 0$ (CO5, L4)

(OR)

b) Let T: X→ Y be a bounded linear operator on a normed space X and let $\lambda \neq 0$. Then prove that the equations $Tx - \lambda x = 0$ and $T^* f - \lambda f = 0$ have the same number of linearly independent solutions. (CO5, L4)

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