PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

22MA4T1 : RINGS AND MODULES
Semester : IV

Course Code	22MA4T1	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2021-22	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision :5 %

Course Objectives : The main objective of the course is to acquire knowledge on Boolean Algebras, isomorphism theorems, Prime ideals in commutative rings, complete ring of quotients, Prime ideal spaces, functional representations of elements of a ring.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	understand the fundamental concepts of rings and modules.
CO2	Understand classical isomorphism theorems and its applications.
CO3	understand the concept of Prime ideals and radicals in commutative rings.
CO4	study the Wedderburn –Artin theorem and its applications.
CO5	study the properties of Prime ideal spaces.

Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO1	**2**	**0**	**0**	**0**	**0**	**0**	**0**
CO2	**0**	**0**	**0**	**0**	**0**	**0**	**3**
CO3	**3**	**0**	**0**	**0**	**0**	**0**	**0**
CO4	**0**	**0**	**0**	**0**	**0**	**0**	**3**
CO5	**2**	**0**	**0**	**0**	**0**	**0**	**0**

UNIT – I

Fundamental Concepts of Algebra: Rings and related Algebraic systems, Subrings, Homomorphisms, Ideals. (Sections 1.1, 1.2 of chapter 1 of prescribed book [1])

UNIT –II

Fundamental Concepts of Algebra: Modules, Direct products and Direct sums, Classical Isomorphism Theorems. (Sections 1.3, 1.4 of chapter 1 of prescribed book [1])

UNIT – III

Selected Topics on Commutative Rings: Prime ideals in commutative Rings, Prime ideals in Special commutative Rings. (Sections 2.1, 2.2 of chapter 2 of prescribed book [1])

UNIT – IV

Selected Topics on Commutative Rings: The Complete Ring of Quotients of a commutative Ring.
(Section 2.3 of chapter 2 of prescribed book [1])

UNIT – V

Selected Topics on Commutative Rings: Rings of quotients of Commutative Semiprime Rings, Prime Ideal Spaces (Sections 2.4, 2.5 of chapter 2 of prescribed book [1])

PRESCRIBED BOOK:

1. Lambek J, Lectures on Rings and Modules, Blaisdell Publications (2009).

REFERENCE BOOKS:

1. Hungerford Thomas W, Algebra, Springer publications (1974).

Course has Focus on : Foundation

Websites of Interest: 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

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M.Sc. Mathematics

Fourth Semester

22MA4T1 - RINGS AND MODULES

Time:3 hours **Max. Marks: 70**

SECTION - A

Answer all questions. **(5x4=20)**

1 a) Prove that in a Boolean algebra, $(a^1)^1 = a$. (CO1, L1)

(OR)

b) Define congruence relation, homomorphic relation and transitive relation. (CO1, L1)

2 a) Define right R-module and left R-module with examples. (CO2, L2)

(OR)

b) Show that a module is Noetherian if and only if every submodule is finitely generated. (CO2, L2)

3 a) Prove that every maximal ideal in a commutative ring is prime. (CO3, L2)

(OR)

b) Prove that every commutative regular ring is semiprimitive. (CO3, L2)

4 a) If D and D^1 are dense ideals of a ring R, show that $D \cup D^1$ is also dense. (CO4, L2)

(OR)

b) Define complete ring of quotients. (CO4, L2)

5 a) Define a compact Topological space and regular open set. (CO5, L1)

(OR)

b) Define interior of a set and exterior of a set V in a topological space Π. (CO5, L1)

SECTION B

Answer all questions. All questions carry equal marks. **(5X10=50)**

6 (a) Show that a Boolean algebra becomes a complemented distributive lattice by defining $a \vee b = (a' \wedge b')'$ & $1 = 0'$ and conversely, any complemented distributive lattice is a Boolean algebra in which these equations are provable identities. (CO1, L3)

(OR)

(b) Show that there is a one-one correspondence between the ideals K and the congruence relations θ of a ring R such that $r - r' \varepsilon K \Leftrightarrow r \; \theta \; r'$ and this is an isomorphism between the lattice of ideals and the lattice of congruence relations.

(CO1, L3)

7 (a) Show that the following statements are equivalent.

(i) R is isomorphic to a finite direct product of rings R_i (i =1,2, …n)

(ii)There exist central orthogonal idempotents e_i ε R such that $1=\sum_{i=1}^{n} e_i$, $e_i R \cong R_i$

(iii) R is a finite direct sum of ideals $K_i \cong R_i$

(CO2, L3)

(OR)

(b) Let B be a sub module of A_R . Then show that A is Artinian if and only if B and A/B are Artinian. (CO2, L3)

8 (a) Show that the radical of a ring R consists of all elements r ε R such that 1 – r x is a unit for all x ε R. (CO3, L3)

(OR)

(b) Let R be a subdirectly irreducible commutative ring with smallest nonzero ideal J. Then show that

(i) The annihilator J* of J is the set of all zero divisors.

(ii) J* is a maximal ideal and J** =J. (CO3, L3)

9 (a) If R is any commutative ring , then show that Q(R) is rationally complete.(CO4, L4)

(OR)

(b) If R is commutative ring, then show that Q(R) is regular if and only if R is semiprime. (CO4, L4)

10 (a) Let Π be any set of prime ideals of the commutative ring R. Then show that Π becomes a topological space where $\rceil A = \{P \in \prod \mid A \not\subset P\}$ (CO5, L3)

(OR)

(b) Show that a Boolean algebra is isomorphic to the algebra of all subsets of a set if and only if it is complete and atomic. (CO5, L3)

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